

Comment on
“Single-inclusive jet production in electron-nucleon collisions
through next-to-next-to-leading order in perturbative QCD”
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Abstract

In the cross section for single-inclusive jet production in electron-nucleon collisions, the distribution of a quark in an electron appears at next-to-next-to-leading order. The numerical calculations in Ref. [1] were carried out using a perturbative approximation for the distribution of a quark in an electron. We point out that that distribution receives nonperturbative QCD contributions that invalidate the perturbative approximation. Those nonperturbative effects enter into cross sections for hard-scattering processes through resolved-electron contributions and can be taken into account by determining the distribution of a quark in an electron phenomenologically.

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In Ref. [1], the cross section for single-jet inclusive production in lepton-nucleon collisions is computed through next-to-next-to-leading order in perturbative quantum chromodynamics (QCD). That computation advances significantly the potential for precision comparisons between theory and experiment for this process. The cross section contains a contribution that is proportional to the distribution of a quark in a lepton, namely, $f_{q/l}(\xi, \mu^2)$, where ξ is the light-cone momentum fraction of the quark and μ is the renormalization scale. Such a contribution could be termed a “resolved-lepton” contribution. The distribution that was used in Ref. [1] is

$$f_{q/l}(\xi, \mu^2) = e_q^2 \left(\frac{\alpha}{2\pi} \right)^2 \left\{ \left[\frac{(1-\xi)(4+7\xi+4\xi^2)}{6\xi} + (1+\xi) \log \xi \right] \log^2 \frac{\mu^2}{m_l^2} + \left[-\frac{(1-\xi)(2+5\xi-2\xi^2)}{\xi} - \frac{8+15\xi-3\xi^2-8\xi^3}{3\xi} \log \xi - 3(1+\xi) \log^2 \xi \right] \log \frac{\mu^2}{m_l^2} \right\}, \quad (1)$$

where m_l is the lepton mass, e_q is the electric charge of the quark, and α is the quantum-electrodynamics (QED) coupling constant. The single and double logarithms of μ cancel the μ -dependence of other factors in the cross section at order $\alpha^2 \alpha_s^2$.

In Ref. [1], $f_{q/l}(\xi, \mu^2)$ is derived by making use of the Dokshitzer, Gribov, Lipatov, Altarelli, Parisi (DGLAP) evolution equation [2–5] in the form

$$\mu^2 \frac{\partial}{\partial \mu^2} f_{q/l} = P_{q\gamma} \otimes f_{\gamma/l} + P_{ql} \otimes f_{l/l}. \quad (2)$$

Here, $f_{\gamma/l}(\xi, \mu^2)$ is the distribution of a photon in a lepton, $f_{l/l}(\xi, \mu^2)$ is the distribution of a lepton in a lepton, $P_{q\gamma}(z)$ and $P_{ql}(z)$ are the DGLAP splitting functions, and \otimes denotes the convolution

$$[P \otimes f](\xi) = \int_{\xi}^1 \frac{dz}{z} P(\xi) f(\xi/z). \quad (3)$$

(In Eq. (2), we have absorbed factors of α into the definitions of the splitting functions.) In Ref. [1], the splitting functions are evaluated to order α and order α^2 , respectively, and the QED distributions on the right side of Eq. (2) are evaluated at leading order in α : $f_{\gamma/l}(\xi, \mu^2)$ is the Weizsäcker-Williams distribution, and $f_{l/l}(\xi) = \delta(1-\xi)$. The distribution in Eq. (1) is obtained by integrating Eq. (2) with the boundary condition $f_{q/l}(\xi, m_l^2) = 0$.

In this comment, we point out that $f_{q/l}(\xi, \mu^2)$ receives nonperturbative QCD contributions that invalidate the expression for the distribution of a quark in an electron defined by Eq. (1). If the lepton has a sufficiently large mass, as is the case for the τ lepton, then $f_{q/l}(\xi, m_l^2)$ can be computed in QCD perturbation theory, and it can be evolved perturbatively from

the scale m_l^2 to the scale μ^2 in order to absorb logarithms of μ^2/m_l^2 into $f_{q/l}(\xi, \mu^2)$. In this case, the expression in Eq. (1) is a valid approximation for $f_{q/l}(\xi, \mu^2)$ in that it captures the logarithmic contributions at leading-order in α .¹ However, when the lepton is an electron or a muon, $f_{q/l}(\xi, \mu^2)$ cannot be computed in QCD perturbation theory.

The nonperturbative nature of $f_{q/l}(\xi, \mu^2)$ can be seen by considering its DGLAP evolution. When one considers QCD corrections, the evolution equation for $f_{q/l}(\xi, \mu^2)$ contains additional contributions that arise from the emission of real and virtual gluons by the quark:

$$\mu^2 \frac{\partial}{\partial \mu^2} \begin{pmatrix} f_{q_i/l} \\ f_g \end{pmatrix} = \begin{pmatrix} P_{q_i\gamma} \otimes f_{\gamma/l} \\ 0 \end{pmatrix} + \begin{pmatrix} P_{q_i l} \otimes f_{l/l} \\ 0 \end{pmatrix} + \sum_{q_j} \begin{pmatrix} P_{q_i q_j} & 2P_{q_i g} \\ P_{g q_j} & P_{g g} \end{pmatrix} \otimes \begin{pmatrix} f_{q_j/l} \\ f_{g/l} \end{pmatrix}, \quad (4)$$

where the sum over q_j includes both quarks and antiquarks. Suppose that one were to follow the procedure in Ref. [1], evolving $f_{q/l}$ from the scale m_l to a hard-scattering scale. The splitting functions in Eq. (4) depend on α_s at scales μ that range from m_l to the hard-scattering scale. If μ is sufficiently large, then the splitting functions can be computed in perturbation theory. However, if μ is less than a scale of order Λ_{QCD} , then the perturbation expansion for the splitting functions fails, and the evolution of $f_{q/l}$ receives nonperturbative contributions. In the case of the electron or the muon, the range of μ includes a region in which perturbative QCD fails and nonperturbative effects dominate.

Although the computation of the short-distance part of the cross section through the order of interest in Ref. [1] requires only that collinear poles through order α^2 be absorbed into $f_{q/l}(\xi, \mu^2)$, a reliable calculation of $f_{q/l}(\xi, \mu^2)$ requires that QCD corrections be taken into account. The concept that the short-distance part of the cross section can be computed at a fixed order in α_s , while the parton distributions, when they are nonperturbative, cannot is, of course, familiar from other hard-scattering processes, such as deep-inelastic scattering.

The nonperturbative distribution for a quark in an electron $f_{q/e}(\xi, \mu^2)$ at a scale μ^2 that is in the perturbative regime of QCD could, in principle, be determined phenomenologically by fitting cross-section predictions to data. A process that is particularly sensitive to $f_{q/e}(\xi, \mu^2)$ is single-inclusive jet production in electron-electron scattering. Alternatively, with some sacrifice of sensitivity, one could make use of cross sections for single-jet inclusive production in electron-nucleon collisions. Lattice calculations might also provide informa-

¹ We note that the expression in Eq. (1) omits constant terms that arise in standard renormalization schemes, such as modified minimal subtraction.

tion on $f_{q/e}(\xi, \mu^2)$. Once the nonperturbative distribution for a quark in an electron has been determined, it could be used to make reliable predictions for the resolved-electron contributions to hard-scattering processes.

Because of the sensitivity of $f_{q/e}(\xi, \mu^2)$ to nonperturbative QCD effects, the expression in Eq. (1) can at best be regarded as a model for the distribution. One unphysical aspect of this model is its double-logarithmic dependence on the electron mass. There is a logarithm of m_e^2 in the Weizsäcker-Williams distribution $f_{\gamma/e}(\xi, \mu^2)$. A second logarithm arises when one integrates Eq. (2) from m_e^2 to μ^2 using the perturbative expressions for the splitting functions. This procedure implies that quarks in the electron are generated by perturbative evolution all the way down to virtualities of order m_e^2 . One would not expect a probe with a virtuality that is much less than a typical hadronic scale to be able to resolve the hadronic structure of the electron. For the range of μ that is considered in Ref. [1], much of the large coefficient $\log^2(\mu^2/m_e^2)$ in Eq. (1) comes from integration over virtualities that are smaller than a typical hadronic scale of, say, 700 MeV. This feature of the model in Eq. (1) would tend to produce a significant overestimate of the contribution from quarks in the electron to the cross section for single-jet inclusive production in electron-nucleon collisions. Other nonperturbative effects that are not accounted for in the model could be substantial, as well.

We note that a sensitivity to nonperturbative QCD effects arises in the same way in the case of the distribution of a quark in a real photon $f_{q/\gamma}$. In this case, the leading-order QED expression for the logarithmic contribution to the distribution that is analogous to Eq. (1) is

$$f_{q/\gamma}(\xi, \mu^2) = e_q^2 \frac{\alpha}{2\pi} [\xi^2 + (1 - \xi)^2] \log \frac{\mu^2}{m_\gamma^2}. \quad (5)$$

The inadequacy of this leading-order logarithmic approximation is manifest in the logarithm of the photon mass m_γ . Of course, it is well established that the distribution of a quark in a real photon involves contributions that cannot be calculated in perturbation theory, but must, instead, be obtained from fits to experimental data. (See, for example, Refs. [6–8].)

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- [1] G. Abelof, R. Boughezal, X. Liu and F. Petriello, Phys. Lett. B **763**, 52 (2016)
<http://dx.doi.org/10.1016/j.physletb.2016.10.022> [arXiv:1607.04921].
- [2] V.N. Gribov and L.N. Lipatov, Sov. J. Nucl. Phys. **15**, 438 (1972) [Yad. Fiz. **15**, 781 (1972)].
- [3] L.N. Lipatov, Sov. J. Nucl. Phys. **20**, 94 (1975) [Yad. Fiz. **20**, 181 (1974)].
- [4] Y.L. Dokshitzer, Sov. Phys. JETP **46**, 641 (1977) [Zh. Eksp. Teor. Fiz. **73**, 1216 (1977)].
- [5] G. Altarelli and G. Parisi, Nucl. Phys. B **126**, 298 (1977).
- [6] F. Cornet, P. Jankowski and M. Krawczyk, Phys. Rev. D **70**, 093004 (2004)
<http://dx.doi.org/10.1103/PhysRevD.70.093004> [hep-ph/0404063].
- [7] P. Aurenche, M. Fontannaz and J. P. Guillet, Eur. Phys. J. C **44**, 395 (2005)
<http://dx.doi.org/10.1140/epjc/s2005-02355-1> [hep-ph/0503259].
- [8] C. Berger, J. Mod. Phys. **6** 1023 <http://dx.doi.org/10.4236/jmp.2015.68107> [arXiv:1404.3551 [hep-ph]].